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13. ABSTRACT *(Maximum 200 words)*

During our previous work, we had established a very dose conncction between polynomial approximation and approximation by neural networks. In fact, we had developed a unified theory of the approximation properties of neural networks, radial basis function (RBF) networks, and generalized regularization networks. Our networks provided an optimal approximation to a class of functions, where the only known a priori assumption was the number of continuous derivatives. The networks did not require ally training in the classical sense, but were given explicitly in terms of the coefficients of the target function in certain orthogonal expansions. Our current objectives are the following.
Modify the formulas for the networks, so that the networks can be obtained in terms of the values of the target function at judiciously chosen points. Develop polynomial wavelets with an eventual objective of integrating these with the theory of "generalized translation networks" which we had previously developed.

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Final Report
Wavelets and neural networks
AFOSR grant number: F49620-97-1-0211
April 1, 1997 – December 31, 1999

1. Objectives

The research supported by this grant is a continuation of our research, supported by AFOSR grant F49620-93-1-0150 (February 15, 1993- July 15, 1996), regarding the approximation properties of neural networks. We had noticed several similarities in the theoretical aspects of wavelets and neural networks. The objective of the current project is to investigate these similarities in further detail.

2. Background

During our previous work, we had established a very close connection between polynomial approximation and approximation by neural networks. In fact, we had developed a unified theory of the approximation properties of neural networks, radial basis function (RBF) networks, and generalized regularization networks. Our networks provided an optimal approximation to a class of functions, where the only known a priori assumption was the number of continuous derivatives. The networks did not require any training in the classical sense, but were given explicitly in terms of the coefficients of the target function in certain orthogonal expansions. Our current objectives are the following.

- Modify the formulas for the networks, so that the networks can be obtained in terms of the values of the target function at judiciously chosen points.
- Develop polynomial wavelets with an eventual objective of integrating these with the theory of "generalized translation networks" which we had previously developed.

3. Accomplishments

The research on this project resulted in 13 publications. In addition, I gave 23 invited lectures in many different countries.

In order to accomplish the objective (a) above, we introduced in [1] certain polynomial operators, similar to the well known de la Vallee Poussin operators in the theory of trigonometric approximation. In addition to the properties of the classical operators, these have the added advantage that their computation involves only the values of the function at the nodes of classical orthogonal polynomials, in particular, the Jacobi and Freud polynomials. In [1] and [2], we studied the boundedness of these operators in various norms. An interesting application of these bounds is that the supremum norm of a polynomial over a continuous interval is shown to be of the same order of magnitude as the maximum absolute value of this polynomial at these zeros.

As an application of this theory and our previous work, we demonstrated in [3] how generalized translation networks can be developed to yield an optimal approximation to functions in the Sobolev classes, knowing only the values of the function at nodes obtained from the zeros of orthogonal polynomials. From a different point of view, these nodes provide the "correct" choice of nodes for universal approximation of a Sobolev class in active learning environment.

In [4], we report on our experiments on the calibration of a degraded phased array antenna using my previous ideas on the theory of neural networks for function approximation. In many situations, the experiments reflect a 50% to 100% improvement on prior work conducted by Southall et. al. at Hanscom Air Force Base.

Next, we reformulated the problem of multiple source direction finding using phased array antennas as the problem of finding the location of the discontinuities of certain derivatives (singularities of different orders) of a function, given its coefficients in the Fourier or Chebyshev polynomial expansion. The classical, compactly supported wavelets are not suitable for this task, partly because their construction requires the knowledge of the corresponding wavelet coefficients, and partly because of inherent theoretical limitations.

In [5], we gave a very general construction of a large class of such polynomial frames which utilize directly the Chebyshev coefficients, and are free of the above limitations. Our frames can be defined using an arbitrary system of orthogonal polynomials. The frame bounds in the least squares sense are independent of the system of orthogonal polynomials used. In the case of Jacobi polynomials (Chebyshev polynomials in particular), our results in [1], [2] are used to establish frame bounds also in the uniform and other norms.

The available multiplier theorems do not apply in this case; new ideas were essential. Our polynomial frames are able to simultaneously detect singularities of different orders. We have given a precise quantitative description connecting the "largeness" of the frame coefficients and the order and location of the singularities. Applied to the multiple source direction finding problem, our methods appear to be remarkably stable under noise.

In [6], we used complex analytic techniques to obtain yet another polynomial operator for the detection of singularities in a function, given its Chebyshev coefficients. Although these operators are not frame operators, they have much sharper localization properties; their values decay exponentially rapidly as we move away from the singularities.

During our research on polynomial frames, we became aware of several other applications of the theme which we were developing. In [7], we carried out a very general study of periodic convolution transforms for the detection of singularities of a periodic function, knowing finitely many of its Fourier coefficients. Our theory enables us to compare a variety of known periodic wavelets, as well as to construct new frames and wavelets with different desired properties. We also constructed "build-up" frames, based on random samples of the target functions. In classical wavelet theory, one starts with a large number of data, and compresses this information into a small number of large wavelet coefficients. Our objective in the construction of the build-up frames is to start with a small number of samples, and then enlarge this data as needed to build higher and higher degree frames, until the detection of singularities is accomplished with a desired accuracy. As before, the same theory can also be used to modify our earlier construction of neural networks so as to utilize incremental amounts of random samples to increase the size of the network as needed to achieve a desired order of approximation.

In [8], we constructed build-up frames based on a discrete orthogonal polynomial system. This construction was motivated by applications in quantum mechanics, pointed out to us by a Russian physicist.

We next turned our attention to the approximation of zonal function networks on the sphere. Our objective was to give results analogous to our constructions of neural networks, but the form of the neural networks is different for the sphere: one does not allow thresholds, and the weights are limited to the sphere. In [10], we constructed zonal function networks that provide an optimal order of approximation to a large class of function spaces defined in terms of the Beltrami operator on the sphere. We have given explicit formulas to "train" these networks using scattered data on the sphere in a coordinate-free manner.

The paper [9] develops certain important and technical quadrature formulas based on scattered data on the sphere. These quadrature formulas play a crucial role in [10] and [7].

The paper [11] is an invited survey paper on our work on neural networks. A more extended survey is given in a five lecture invited tutorial we gave during an international conference on wavelets and related topics in Delhi, India [13]. The paper [12] surveys our work and other related work on the zonal function network approximation on the sphere.

4. Personnel supported

Hrushikesh N. Mhaskar, PI.

5. Technical Publications

1. *Bounded quasi-interpolatory polynomial operators,* Journal of Approximation Theory, **96** (1999), 67–85. (With J. Prestin).

2. *On Marcinkiewicz-Zygmund-type inequalities,* in "Approximation theory: in memory of A. K. Varma", (N. K. Govil, R. N. Mohapatra, Z. Nashed, A. Sharma, and J. Szabados Eds.), Marcel Dekker, 1998, pp.389–404. (With J. Prestin).

3. *On a choice of sampling nodes for optimal approximation of smooth functions by generalized translation networks,* in "Artificial Neural Networks, Conference Publication No. 440" (IEE), 1997, pp. 210-215. (With J. Prestin).

4. *Neural beam-steering and direction finding,* in "Neural Networks in Engineering Systems", (A. B. Bulsari and S. Kallio Eds.), Royal Institute of Technology, Stockholm, 1997, pp. 269-272. (With H. Southall).

5. *Polynomial frames for the detection of singularities,* in "Wavelet Analysis and Multiresolution Methods" (Ed. Tian-Xiao He), Lecture Notes in Pure and Applied Mathematics, Vol. 212, Marcel Decker, 2000, 273–298. (With J. Prestin).

6. *On a sequence of fast decreasing polynomial operators,* in "Applications and Computation of Orthogonal Polynomials" (Eds. W. Gautschi, G.H. Golub, G. Opfer) Internat. Ser. Numer. Math., Birkhäuser, Basel, 1999, 165-178. (With J. Prestin).

7. *On the detection of singularities of a periodic function*, Advances in Computational Mathematics, **12** (2000), 95–131 (With J. Prestin).

8. *On a build-up polynomial frame for the detection of singularities*, in "Self-Similar Systems" (V. B. Priezzhev and V. P. Spiridonov Eds.), Joint Institute for Nuclear Research, Dubna, Russia, 1999, pp. 98–109. (With J. Prestin).

9. *Quadrature Formulas on Spheres Using Scattered Data*, To appear in Mathematics of Computation. (With F. J. Narcowich and J. D. Ward).

10. *Approximation Properties of Zonal Function Networks Using Scattered Data on the Sphere*, Advances in Computational Mathematics, **11** (1999), 121–137 (With F. J. Narcowich and J. D. Ward).

11. *Approximation of smooth functions by neural networks*, in "Dealing with complexity: A neural network approach", (K. Warwick et. al. eds), "Perspectives in Neural Computing", Springer Verlag, London, 1998, pp.189–204.

12. *Representing and analyzing scattered data on the sphere*, To appear in "Multivariate approximation and applications" (A. Pinkus, D. Leviatan, N. Dyn, and D. Levin Eds.), Cambridge University Press, Cambridge. (With F. J. Narcowich and J. D. Ward).

13. *Approximation theory and neural networks*, accepted for publication in "Wavelet Analysis and Applications, Proceedings of the international workshop in Delhi, 1999" (P. K. Jain, M. Krishnan, H. N. Mhaskar J. Prestin, and D. Singh Eds.), Narosa Publishing, New Delhi, India.

6. Interactions

I gave the following invited lectures as indicated.

1. Ohio State University, Columbus, Ohio, April 1997.

2. Oak Ridge National Laboratory, Oak Ridge, Tennessee, April 1997.

3. University of Kentucky, Lexington, Kentucky, April 1997.

4. Technishe Universitatt, Munich, Germany, July 1977.

5. Katholische Universitatt, Eichstaett, Germany, July 1977.

6. International symposium on Approximation Theory, Nashville, Tennessee, January, 1998. (Also organized and chaired a special session on neural networks.)

7. Oberwolfach, Germany, March, 1998.
8. Montecateni, Italy, June, 1998.
9. Eichstaett, Germany, June, 1998.
10. Dortmund, Germany, July, 1998.
11. Hohenheim, Germany, July, 1998.
12. Goettingen, Germany, July, 1998.
13. Ukrainian Academy of Sciences, Kiev, Ukraine, July, 1998.
14. International Symposium on Self-similar systems, Dubna, Russia, August, 1998.
15. International Conference on signal conference, Eilat, Israel, September, 1998.
16. Special session on Wavelet Analysis, American Mathematical Society, Urbana-Champaign, Illinois, March, 1999.
17. Air Force Institute of Technology, Dayton, Ohio, March, 1999.
18. International Conference honoring R. S. Varga, Kent, Ohio, March, 1999.
19. International Workshop on Orthogonal Polynomials, Ballenstaedt, Germany, April, 1999.
20. International Conference on Approximation Theory, Kiev, Ukraine, May, 1999.
21. International Conference on Computational Mathematics, Oxford, U.K., July, 1999.
22. Aston University, Birmingham, U. K., July, 1999.
23. International Workshop on Wavelet Analysis and related topics, Delhi, India, August, 1999.

7. Honor

Fellowship, Alexander von Humboldt Foundation, May-July 1998.

8. Patent disclosures

None